1. INTRODUCTION

In 2006, Pendry et al. [1] and Leonhardt [2] independently introduced the so-called method of transformation optics (TO), which relies on the form-invariance of 3-vector Maxwell’s equations under a spatial transformation. The invariance of Maxwell’s equations implies that the permittivity and permeability tensors must be adjusted to comply with the topological constraints of the system. TO provides a systematic mechanism to make these adjustments based on the spatial transformations adopted [2,4]. The first experimental demonstration of the TO-based device was a cylindrical invisibility cloak at microwave frequency, which was fabricated with the arrays of split-ring resonators [5]. Following this pioneering work, a number of other electromagnetic devices—including electromagnetic concentrators [6], field rotators [7], enhanced scatterers [8], and even optical-illusion devices [9]—have been reported under the theoretical framework of TO. Moreover, the application of TO was soon extended from optics [10] to acoustics [11,12] and elastics [13].

Most recently, it has been proposed that the method of TO can even be applied to plasmonics, in order to manipulate the power flow of surface plasmon polaritons (SPPs) along metal–dielectric interfaces [14–18]. The manipulation may be achieved by varying the permittivity and permeability of the materials making up the interface using TO-based mappings [19–23]. By doing this, one needs to exercise a certain degree of care due to some particular features peculiar to SPPs. For example, even a small abrupt discontinuity in the material’s constitutive parameters or geometries will result in radiation scattering, which significantly reduces the effectiveness of the plasmonic devices [24,25]. Since metamaterials obtained using transformations that lead to the desired functionalities in the most straightforward manner are usually highly anisotropic, inhomogeneous, and even singular, their realization in the foreseeable future seems to be impractical. It is not surprising therefore that great efforts have been devoted to simplify the constitutive parameters of metamaterials to a feasible level. One of the ways to achieve such simplification is to apply linear transformations [26–28], for which all coordinate mappings from the original space to the physical space are linear. The resulting metamaterials will be characterized by solely homogeneous constitutive parameters. This simplification technique was used to experimentally demonstrate macroscopic, wide-band cloaks for visible light, with a great potential for practical applications [29–31].

Although the method of linear transformation is widely employed in electromagnetics and acoustics, its application to plasmonics has not been considered so far. In this paper, by utilizing linear transformations, we demonstrate that SPPs can be efficiently manipulated at metal–dielectric interfaces with solely homogenous metamaterials. In Section 3, we demonstrate how surface plasmon modes can be tightly guided around the trapezoid surfaces without scattering losses, while in Section 4 we design a plasmonic cloak of invisibility. In Section 5, we also show that homogeneous metamaterials obtained with linear transformations may be used to enhance the energy of SPPs in the device called “plasmonic concentrator.” Full-wave simulations of the considered designs illustrate the validity of the methodology employed. Our study clearly demonstrates the possibility of novel applications of homogenous metamaterials obtained using TO in various types of plasmonic systems, which can be potentially fabricated due to the simplified constituent parameters.
dielectric constituents of the system [18]. However, we will not significantly affect the performance of the TO-based devices by not transforming the area occupied by metal, because the energy of SPPs predominantly resides inside the dielectric [14,15]. With this in mind, consider a linear transformation

\[
\begin{align*}
    x' &= a_1 x + b_1 y + c_1 z + d_1, \\
    y' &= a_2 x + b_2 y + c_2 z + d_2, \\
    z' &= a_3 x + b_3 y + c_3 z + d_3,
\end{align*}
\]

which maps the background space \((x, y, z)\) that is filled with a homogeneous, isotropic dielectric characterized by the tensors \(\epsilon = \epsilon_0 I\) and \(\mu = \mu_0 I\) (\(I\) is the identity matrix) to a new physical space \((x', y', z')\). According to the TO theory (see, for example, Pendry et al. [1]), the constitutive parameters of the transformed medium may be calculated as follows:

\[
\begin{align*}
    \epsilon' &= \frac{J_{\epsilon} J^T}{\det J} \epsilon_0 M, \\
    \mu' &= \frac{J_{\mu} J^T}{\det J} \mu_0 M,
\end{align*}
\]

where \(J\) is the Jacobian of the mapping, \(M\) is the matrix with components \(M_{ij} = a_i b_j + b_i a_j + c_i c_j\), and

\[
\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.
\]

As seen from Eq. (2), the linear transformation of the originally homogeneous dielectric leads to a new medium with spatially independent permittivity \(\epsilon'\) and permeability \(\mu'\). This implies that the TO-based devices, designed using linear transformations, can be made of only homogeneous metamaterials.

In the next sections, we consider the three types of TO applications in plasmonics: (i) assistance in guidance SPPs along irregular interfaces, (ii) cloaking objects on the surface of a metal, and (iii) concentration of SPP energy within a given area. We illustrate the performance of the transformation media in each case via numerical simulations, which are performed by a commercial COMSOL Multiphysics software (version 4.2). In the simulations, we assume that SPPs propagate along air–silver interfaces and take \(\epsilon_0 = \mu_0 = 1\). The dispersion of silver permittivity is described by Drude model

\[
\epsilon_{\omega}(a) = \epsilon_\infty - \frac{\alpha_p^2}{\omega(\omega + i\gamma)},
\]

with the following set of parameters [32]: \(\epsilon_\infty = 6\), \(\alpha_p = 1.5 \times 10^{10}\) rad/s, and \(\gamma = 7.7 \times 10^{15}\) rad/s.

3. PLASMONIC GUIDER

We start by considering the plasmonic wave traveling along an interface with a trapezoid protrusion shown in Fig. 1(a). For the sake of simplicity, we restrict ourselves to the two-dimensional (2D) scenario assuming that our structure is homogeneous in the \(z\) direction. In order to eliminate the scattering of SPPs at the protrusion using homogeneous metamaterials, we employ the following linear transformation that maps the large trapezium (with bases at \(y = 0\) and \(y = h_2\)) in Fig. 1(a) to the irregular region shaded in orange:

\[
\begin{align*}
    x' &= x, \\
    z' &= z.
\end{align*}
\]

Here, \(a, b, h_1\), and \(h_2\) are the dimensions of the regions in Fig. 1(a). Since before the transformation the large trapezium was filled with air and the metallic surface was flat, SPPs will smoothly bend around the new boundary, provided that the orange region is filled with the artificial medium that is obtained by transforming the air region using Eq. (4).

The use of Eqs. (2)-(4) yields the following constitutive parameters for the artificial medium:

\[
\begin{align*}
    \epsilon' &= \mu', \\
    \epsilon' &= \begin{cases}
    \frac{h_2}{h_2-h_1} y + \frac{h_1 y}{h_2} h_2, & a < |x| < b, \\
    0, & |x| \leq a,
\end{cases}
\end{align*}
\]

where \(\text{sgn}(\cdot)\) is the sign function. Since SPPs propagate in the form of transverse-magnetic (TM) modes, only the \(x\) and \(y\) components of the electric field and the \(z\) component of the magnetic field are nonzero. This implies that the values of the tensor components \(\epsilon'_{zz}, \mu'_{zz}, \mu'_{yy}, \mu'_{yy},\) and \(\mu'_{zz}\) do not affect propagation of the surface wave. As a result, the metamaterial parameters can be reduced to

\[
\begin{align*}
    \epsilon' &= \begin{cases}
    \frac{h_2}{h_2-h_1} y + \frac{h_1 y}{h_2} h_2, & a < |x| < b, \\
    0, & |x| \leq a,
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\begin{align*}
    \epsilon' &= \begin{cases}
    \frac{h_2}{h_2-h_1} y + \frac{h_1 y}{h_2} h_2, & a < |x| < b, \\
    0, & |x| \leq a,
\end{cases}
\end{align*}
\]
and $\mu' = h_2'(h_2 - h_1)$. It is worth noting that the obtained metamaterial parameters may also be used to assist smooth SPP mode routing across the trapezoid groove shown in Fig. 1(b). In this case, the linear transformation stretches the background space inside the trapezoid of height $h_2$ (located at $y > 0$) to form the groove of depth $h_1$.

Figure 2 shows how the magnetic field pattern of the plasmon mode traveling along the two types of irregular interfaces changes in the presence of the appropriately designed homogeneous metamaterials. It was assumed that the free-space wavelength is 900 nm, which corresponds to the SPP propagation length of about 150 $\mu$m. One can see that the propagation across either the protrusion or the groove leads to a drastic loss of SPP energy in the absence of the transformation media. For example, more than 90% of the mode energy is irradiated to the far field in the case of the protrusion shown in Fig. 2(a). Although the radiative loss is a bit smaller in Fig. 2(b) due to the relatively small width of the groove, the groove still significantly distorts the field pattern and eventually destroys the guided mode. When the metamaterials designed using Eq. (6) are placed on the top of the bump or inside the groove [see Figs. 2(c) and 2(d)], the propagation picture changes completely. Now, the SPP mode is efficiently guided through the surface irregularities and preserves its field pattern behind the obstacles, with less than 3% of its energy lost to radiation in either of the cases. The small energy loss is predominantly due to the error resulting from not substituting metal with the corresponding metamaterial, as was discussed above.

4. PLASMONIC INVISIBILITY CLOAK

Another possible application of homogeneous metamaterials obtained through linear transformations lies in the suppression of SPP scattering at compact obstacles on metal-dielectric interfaces. The suppression may be achieved by surrounding the obstacles with metamaterials that steer SPPs around them to prevent the scattering. In other words, the obstacles become almost undetectable via the surface plasmon waves, as if they were covered by a plasmonic invisibility cloak. Here, we focus on the invisibility cloak in the form of a square shield, which is similar to that used by Li et al. [26] in order to render objects invisible when probed with microwave radiation.

The transformation leading to the square cloak involves two steps illustrated by Fig. 3. On the first step, the line segment of length $2a$ in the background space—presented by a square of area $2b^2$ in Fig. 3(a)—is stretched such that its length becomes $2c$. As a result of this, the space surrounding the segment is partially stretched [orange rhombus in Fig. 3(a)] and partially compressed (gray regions). On the second step, the segment of length $2c$ is expanded to a square region in the physical space, which is shaded in green in Fig. 3(b). The coordinate mapping of this transformation is

$$x' = \frac{b - c}{b - a} x + \frac{c - a}{b - a} (b - y) \text{sgn}(xy), \quad y' = y, \quad z' = z \quad (7)$$

for the gray region and

$$x' = \frac{c}{a} x, \quad y' = \varepsilon \left( \frac{|x|}{a} - 1 \right) \text{sgn} y + \frac{b - c}{b} y, \quad z' = z \quad (8)$$

for the orange region. The permittivity and permeability tensors of metamaterials required to build the cloak are found using Eqs. (2) and (3) to be

$$\varepsilon' = \mu' = \begin{pmatrix} \frac{(c - a)^2 + (c - b)^2}{(a - b)(c - b)} & \text{sgn}(xy) & \varepsilon = \varepsilon_b \\ 0 & \varepsilon \varepsilon_b & 0 \\ 0 & 0 & \frac{a - b}{c - b} \end{pmatrix}$$

(9)

in the gray region and

$$\varepsilon' = \mu' = \begin{pmatrix} \frac{bc}{a(b - c)} & \text{sgn}(xy) & \frac{bc}{a(b - c)} \\ 0 & \frac{bc}{a(b - c)} & 0 \\ 0 & 0 & \frac{ab}{c(b - c)} \end{pmatrix}$$

(10)

in the orange region. These expressions show that $\varepsilon'$ and $\mu'$ are constant in both regions and are free of singularities, provided that $a > 0$. Since the function $\text{sgn}(xy)$ is positive in quadrants I and III and negative in quadrants II and IV, four types of homogeneous metamaterials are needed to build the plasmonic cloak.

It is of significance that if a plane SPP wave of wavelength $\lambda$ falls obliquely onto the cloak, it scatters at the medium filling the hiding region, because the line segment of length $2a$ is seen by the wave as an infinitesimally thin shield made of a perfect electric conductor. The scattering efficiency scales...
as $2\hat{n}/\lambda$, where $2\hat{n}$ is the projection of the segment $2\hat{n}$ onto the wavefront, and peaks for waves propagating parallel to the $y$ axis. In order to reduce the scattering and improve the cloak’s performance, one needs to choose the ratio $2\hat{n}/\lambda$ as small as practicable. The value of $\alpha$, however, cannot be too small, because it causes metamaterials inside the orange region to have large constitutive parameters [see Eq. (10)].

In the ideal situation where the cloak steers around an obstacle the entire field of the guided plasmon mode inside the dielectric, its height in the $z$ direction should be infinite. However, in reality, owing to the strong localization of SPPs to metal–dielectric interfaces, only the heights of several evanescent decay lengths are required. The decay length of SPP fields into the dielectric region is related to the permittivities of metal and dielectric, $\varepsilon_m$ and $\varepsilon_d$, as follows [24]:

$$\delta_d = (\lambda/2\pi)|\sqrt{1 + \varepsilon_m/\varepsilon_d}|/\Im\sqrt{\varepsilon_m - \varepsilon_d}.$$  

Using this formula, we find $\delta_d \approx 3 \mu m$ for the parameters of Fig. 4. It is reasonable therefore to choose the height of the plasmonic cloak to be equal or slightly above $6 \mu m$, in which case it captures more than 98% of the energy of the SPP mode.

Figure 4 shows how a metallic bar of square cross section may be hidden from SPPs using the homogenous metamaterials whose constitutive parameters are given in Eqs. (9) and (10). The bar touches the interface $z = 0$ and its size is equal to the size of the cloak’s hiding region, with height equal to $6 \mu m$. The scattering at the bare metallic bar for the case when SPPs propagate parallel to the $x$ axis is shown in Fig. 4(a). The pattern of the SPP mode is seen to be strongly distorted, with a clearly visible shadow behind the obstacle. When the bar is covered by the cloak, as shown in Fig. 4(b), the electromagnetic field of the mode gets squeezed inside the metamaterials and efficiently bent around the obstacle: the scattering in the forward and backward directions becomes greatly suppressed and the mode recovers its original pattern behind the bar.

It should be noted that the scattering in Fig. 4(b) corresponds to the optimal performance of the plasmonic cloak, because the original segment of length $2a$ is projected into a point for SPPs traveling in the $x$ direction. As a result of this, the cloak exhibits a decent invisibility performance even for a relatively large ratio $2\hat{n}/\lambda = 0.65$. However, since the cloak is not symmetric with respect to the rotation over an angle $\pi/2$ around the $z$ axis, the scattering drastically increases for SPPs incident from the $y$ direction. The scattering may be reduced by employing the cloak with much smaller ratio $2\hat{n}/\lambda$.

5. PLASMONIC CONCENTRATOR

In order to set up an area on the metallic surface that is almost free of electromagnetic fields of SPPs, we had to expand the background space and create a hollow region shown in Fig. 3(b). On the other hand, if the purpose is to enhance the energy density by focusing SPPs to a tiny spot on a surface, one needs to perform a somewhat opposite transformation, which amounts to a partial compression of the background space to a smaller domain. We shall be referring to the devices that enable such focusing as plasmonic concentrators.

Figure 5 shows how to design a simple plasmonic concentrator having a square cross section. Following the two-step mapping procedure outlined in [27], we transform the purple square (of area $2a^2$) in the background space [see Fig. 5(a)] into a smaller one (of area $2c^2$) in the physical space [see Fig. 5(b)]. First, the rhombus with diagonals $2a$ and $2b$ is compressed along the $x$ axis by a factor of $c/a$, and the gray regions are stretched accordingly. The boundary of the resulting region is shown in Fig. 5(a) by dotted lines. On the second step, we compress the dashed rhombus on the same figure along the $y$ direction by a factor of $c/a$, and transform it into a square. The coordinate transformation corresponding to the described mapping procedure is

$$x' = \frac{b - c}{b - a} x + \frac{a - c}{a - b} (b - y) \text{sgn}(xy), \quad y' = y, \quad z' = z \quad (11)$$

for the gray regions,

$$x' = \frac{c}{a} x, \quad y' = \frac{b - c}{b - a} y + \frac{a - c}{a - b} \left(1 - \frac{|x|}{a} \text{sgn}(y)\right), \quad z' = z \quad (12)$$

for the orange regions, and

Fig. 4. (Color online) Snapshots of magnetic field in planes $y = 0$ and $z = 0$ for SPP scattering at (a) metallic bar and (b) metallic bar inside the invisibility cloak. Arrows show the propagation direction of SPPs. The geometric parameters are $a = 0.5 \mu m$, $b = 5 \mu m$, and $c = 3 \mu m$ (see Fig. 3); the wavelength of incident SPPs is $1.55 \mu m$.

Fig. 5. (Color online) Transformation of (a) a square background-space region of area $2b^2$ to (b) plasmonic concentrator with a working area $2c^2$ (shaded in purple). Each region in (a) is mapped to the region of the same color in (b).
\[ x' = (c/a)x, \quad y' = (c/a)y, \quad z' = z \] (13)

for the purple region. With this relations, the metamaterial tensors take the form

\[
e' = \mu' = \begin{pmatrix}
\frac{(a-c)^2 + (b-c)^2}{b(a-b)} & \text{sgn}(xy) \frac{a-c}{b-c} & 0 \\
\text{sgn}(xy) \frac{b-c}{b-c} & 0 & 0 \\
0 & 0 & \frac{b-a}{b-c}
\end{pmatrix}
\] (14)

in the gray region,

\[
e' = \mu' = \begin{pmatrix}
\frac{c(b-a)}{a(b-c)} & \text{sgn}(xy) \frac{b(c-a)}{a(b-c)} & 0 \\
\text{sgn}(xy) \frac{a(c-b)}{a(b-c)} & 0 & 0 \\
0 & 0 & \frac{a(b-a)}{c(b-c)}
\end{pmatrix}
\] (15)

in the orange region, and

\[
e' = \mu' = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & (a/c)^2
\end{pmatrix}
\] (16)

in the purple region. The tensors in Eqs. (14) and (15) are independent on the Cartesian coordinates within four regions constituting the concentrator’s cladding and inside the purple working region, which implies that five types of homogeneous metamaterials are to be used to build the plasmonic concentrator.

In Fig. 6, we show the performance of the plasmonic concentrator composed of homogeneous metamaterials with parameters given in Eqs. (14)–(16). The design of the concentrator suggests that the energy of the SPP mode accumulated over the area of the big purple square in Fig. 5(a) is squeezed into the small purple square in Fig. 5(b). This leads to the enhancement of the mode’s intensity by a factor of \((a/c)^2 = 9\), as compared to the intensity of SPPs outside the concentrator. Notice that, owing to the perfect impedance matching between the metamaterials and surrounding medium, the scattering of the incident wave is absent and its field pattern is fully recovered after the defocusing stage on the right side of the concentrator.

In sharp contrast to the plasmonic cloak discussed in the previous section, the performance of the plasmonic concentrator is isotropic in the \(x-y\) plane and its enhancement factor is independent of the incident angle of SPPs. This is a consequence of the fact that the concentrator was obtained through the isotropic compression of the primordial region in the background space. However, if one compresses the purple square in Fig. 5(a) by different factors along the \(x\) and \(y\) axes, then the enhancement factor will be a function of the SPPs’ propagation direction. Also noteworthy is that by choosing \(a < c\) in the above equations, one may realize a plasmonic expander—the device that dilutes the electromagnetic field of SPPs inside of it, while not affecting the field outside.

As a concluding remark, we would like to stress that the metamaterials employed in the considered three types of plasmonic devices are characterized by both anisotropic permittivity and anisotropic permeability. While many natural dielectrics exhibit anisotropic optical response and this significantly facilitates fabrication of artificial composites with similar characteristics, the anisotropic magnetics are by far less frequently met in nature, especially at optical frequencies. One may lift the requirement that the permeability tensor must be anisotropic by employing the fact that the magnetic field of SPPs is transversely polarized, and thus only one of its components varies during the propagation. We refer the reader to recent papers in [29–31], where the process of simplification of the metamaterial parameters is described in more detail. Using the simplification procedure, it is possible to realize nonideal plasmonic devices based solely on nonmagnetic, uniaxial crystals that can be found in nature.

6. CONCLUSIONS

In this paper, we have applied the method of linear TO to several problems of manipulating the propagation of SPPs along metal–dielectric interfaces. We demonstrated that it is possible to control the flow of SPPs using homogeneous metamaterials, as opposed to the widely accepted prescription, which requires highly inhomogeneous metamaterials. Specifically, we described how to construct a plasmon guider for a particular nonflat surface, an invisibility cloak that renders objects undetectable via SPPs, and a concentrator of the SPPs’ energy. The functionalities of these devices were visualized, and their performance was investigated using finite-element simulations. Our results clearly show that the method of linear transformations is a simple and effective tool, which produces designs for feasible plasmonic devices utilizing homogenous metamaterials.

ACKNOWLEDGMENTS

The work of W. Zhu and M. Premaratne is supported by the Australian Research Council, through its Discovery Grant scheme under grant DP110100713. I. D. Rukhlenko also gratefully acknowledges the Australian Research Council for financial support within the frame of the Discovery Early Career Researcher Award DE120100055.

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